Unsupervised learning: Exercise

1- Simulate the development of ocular dominance using a network with 2 input units (one right, one left) and one output unit. Use the following equation for the learning rule:

$$\delta\mathbf{w} = \mathbf{Q}\cdot\mathbf{w} - \left(\frac{\mathbf{w}\cdot\mathbf{Q}\cdot\mathbf{n}}{N_u}\right)\mathbf{n}$$

$$\mathbf{w}^*(t+1) = \mathbf{w}(t) + \alpha\delta\mathbf{w}(t)$$

$$w_i(t+1) = \begin{cases} 1 & \text{if } 1 < w_i^*(t+1) \\ 0 & \text{if } w_i^*(t+1) < 0 \\ w_i^*(t+1) & \text{if } 0 < w_i^*(t+1) < 1 \end{cases}$$

where N_u is the number of input units (2 in this case), **n** is a column vector of N_u 1's, α is a learning rate (α =1 should work but try different values) and **Q** is equal to:

$$\mathbf{Q} = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

Do the weights lined up with the first or second eigenvector of **Q**? Do you get ocular dominance?

2- same as in 1 but with 101 output units and no lateral connections. Plot the weight pattern. Do you get ocular dominance? Do you get periodic columns? (Note: make sure that the normalization term is local to each output unit, i.e., its value should be specific to each unit on each iteration)

3- same as in 2 but this time, use the lateral connection kernel:

$$K(x) = A\exp\left(-\frac{(x-51)^2}{2s_1^2}\right) - B\exp\left(-\frac{(x-51)^2}{4s_2^2}\right)$$

$$s_1 = 10, s_2 = 20, A = 1, B = 0.5$$

x is expressed in terms of number of units. The learning rule in this case is:

$$\delta\mathbf{W} = \mathbf{KWQ} - \left(\frac{\mathbf{K}\cdot\mathbf{W}\cdot\mathbf{Q}\cdot\mathbf{N}}{N_u}\right)$$

$$\mathbf{W}^*(t+1) = \mathbf{W}(t) + \alpha\delta\mathbf{W}(t)$$

$$w_{ij}^*(t+1) = \begin{cases} 1 & \text{if } 1 < w_{ij}^*(t+1) \\ 0 & \text{if } w_{ij}^*(t+1) < 0 \\ w_{ij}^*(t+1) & \text{if } 0 < w_{ij}^*(t+1) < 1 \end{cases}$$

where **K** is the matrix whose rows are shifted copies of K(x) (make sure to treat x in K(x) as a periodic variable) and **N** is a 2x2 matrix of ones.

Do you get ocular dominance columns?

4- Verify that W and K have the same fundamental frequency.

5- Rerun the simulation with the following parameters:

$$K(x) = A\exp\left(-\frac{(x-51)^2}{2s_1^2}\right) - B\exp\left(-\frac{(x-51)^2}{4s_2^2}\right)$$

$$s_1 = 5, s_2 = 10, A = 1, B = 0.5$$

What do you observe and why?

6- same as in 5 with:

$$K(x) = A\exp\left(-\frac{(x-51)^2}{2s_1^2}\right)$$

$$s_1 = 10, s_2 = 20, A = 1$$